

Spin-dependent fragmentation functions of Gluon splitting into heavy quarkonia considering three different scenarios

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Heavy quarkonium production is a powerful implement to study the strong interaction dynamics and QCD theory. Fragmentation is the dominant production mechanism for heavy quarkonia with large transverse momentum. With the large heavy quark mass, the relative motion of the heavy quark pair inside a heavy quarkonium is effectively nonrelativistic and it is also well-known that their fragmentation functions can be calculated in the perturbative QCD framework. Here, we analytically calculate the process-independent fragmentation functions for a gluon to split into the spin-singlet and -triplet S -wave heavy quarkonia using three different scenarios. We will show that the fragmentation probability of the gluon into the spin-triplet bound-state is the biggest one.

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I. INTRODUCTION

Heavy quarkonia are the simplest particles when the strong interactions are concerned. Studying their production mechanism is important for understanding QCD and the strong interaction dynamics. In theory, it is well-known that the dominant mechanism to produce a heavy quarkonium at large transverse momentum is fragmentation [1] so this mechanism is described by the fragmentation functions (FFs). In fact, the FF shows hadron production probability from the initial high-energy parton. The FFs are universal quantities so that are independent of the initial parton production processes. The particular importance of FFs is for model-independent predictions of the cross sections at the Large Hadron Collider (LHC). Basically, the fragmentation is related to the low-energy part of the hadron production process and forms the non-perturbative aspect of QCD so, in principle, should be extracted from experimental data. In practice, it is hard to extract so many initial FFs from data directly.

In Ref. [2], using the phenomenological approach which is based on the data analysis we propounded a new functional form of pion and kaon FFs up to NLO, obtained through a global fit to the data from the single-inclusive annihilation $e^-e^+ \rightarrow hX$ and also from the semi-inclusive DIS asymmetry data from HERMES and COMPASS.

Since heavy quarkonia have simple internal structures it is well-known that the perturbative QCD approximations to their FFs are well-defined in the nonrelativistic QCD factorization framework [3–5]. The perturbative QCD scheme was first applied by Bjorken [6], Suzuki [7], Amiri and Ji [8], where the best elaborate model is proposed by Suzuki [9] which is based on the convenient Feynman diagrams and the wave function of the respective heavy meson. In this model, the heavy FFs are calcu-

lated using a diagram similar to that in Fig. 2, so the analytical expression of FFs depends on the transverse momentum k_T of the initial parton which appears as a phenomenological parameter (e.g. see Eq. (30)). In Ref. [10], using this model we presented an exact analytical expression of the initial scale FF for c -quark to split into S -wave D^+/D^0 mesons to LO. To check our results, we showed that there is a good consistency between our result and the current well-known phenomenological models and also with the experimental data from BELLE and CLEO. In Ref. [11], using this model authors have calculated the FFs for a heavy quark Q to fragment into the vector $Q\bar{Q}(^3S_1)$ and pseudoscalar $Q\bar{Q}(^1S_0)$ heavy quarkonia to LO. Using the same model, authors in Ref. [12] have computed the fragmentation of a heavy quark Q into a QQQ -system with three identical flavor, i.e. $D_{b \rightarrow \Omega_{bbb}}(z, \mu_0)$ and $D_{c \rightarrow \Omega_{ccc}}(z, \mu_0)$. Here, D stands for the fragmentation function and z is the fragmentation parameter which refers to the energy fraction of the initial parton which is taken away by the detected hadron and μ_0 is the initial factorization scale.

In high energy processes, the large contribution of quarkonium production results from gluon fragmentation [13, 14]. This is confirmed by the comparison between the theoretical predictions and the experimental measurements of the heavy quarkonium cross sections. For this reason, using the Suzuki's model we focus on the gluon fragmentation and drive an analytical form of the FFs for a gluon to fragment into the vector $Q\bar{Q}(^3S_1)$ and pseudoscalar $Q\bar{Q}(^1S_0)$ heavy quarkonia. To impose the spin effects of the heavy quarkonia into the FFs we apply three different scenarios. Comparison of all scenarios shows that the fragmentation probability of the gluon into the spin-triplet quarkonium is the biggest one.

This paper is organized as follows. In Sec. II, we explain our theoretical model to calculate the FFs in detail. We discuss the use of pQCD in calculating the fragmentation of a charm quark into the heavy charmonium H_c . In Sec. III the analytical expressions for the polarized

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and unpolarized FFs in three different scenarios are presented. Next we shall present our numerical results for the gluon FFs. Our conclusion is summarized in Sec. IV.

II. THEORETICAL DETERMINATION OF FFs: PERTURBATIVE QCD MODEL

As is pointed out in Refs. [1, 3] the fragmentation function for hadrons containing a heavy quark Q or a heavy antiquark \bar{Q} can be computed theoretically using perturbative QCD (pQCD). The first theoretical effort to illustrate the hadroproduction procedure by a heavy quark was made by Bjorken [6] by using a naive quark-parton model. He found out that the inclusive distribution of heavy hadron should peak nearly at $z = 1$, where z refers to the scaled energy variable. This property is important for heavy quarks for which the peak of heavy quark FF happens closer to $z = 1$. The pQCD model was followed by Suzuki [7], Amiri and Ji [8].

Here, using the Suzuki's model we focus on the gluon fragmentation function and obtain an analytical form of the gluon fragmentation into a spin-singlet and -triplet S-wave heavy quarkonium by employing three different scenarios. Our results shall be compared with the unpolarized one. To this end, we consider a special example: $g \rightarrow H_c (= c\bar{c})$ for which the respective Feynman diagrams shown in Fig. 1. First, we consider Fig. 2 where the basic process of gluon fragmentation into the charmonium bound state along with the spins and the four-momenta of meson and partons are shown. Following Ref. [9], we adopt the infinite momentum frame where the fragmentation parameter in the usual form $z = (E^H + p_{||}^H)/(E^g + p_{||}^g)$ is reduced to the more popular one as

$$z = \frac{E^H}{E^g}. \quad (1)$$

With the large heavy quark mass, the relative motion of the heavy quark pair inside the quarkonium is effectively nonrelativistic [15], so the squared relative velocity of the heavy quark pair in the quarkonium rest frame is $v^2 \approx 0.22$ for the J/ψ and $v^2 \approx 0.1$ for the Υ [16]. Here, according to the Lepage-Brodsky's approach [17], by neglecting the relative motion of the heavy quark pair inside the quarkonium we assume, for simplicity, that Q and \bar{Q} are emitted collinearly with each other and move along the Z -axes. Indeed, the Fermi motion of the constituent quarks in the bound state is neglected.

In the Suzuki's model, the FF for the production of S-wave bound state is defined as [9, 18]

$$D_g^{H_c}(z, \mu) = \int d^3\bar{\mathbf{P}} d^3\mathbf{k}' |T_M|^2 \delta^3(\mathbf{k}' + \bar{\mathbf{P}} - \mathbf{p}'), \quad (2)$$

where T_M is the probability amplitude of the meson production which is expressed as the convolution of the hard scattering amplitude T_H and the process-independent distribution amplitude Φ_M . The hard amplitude T_H can

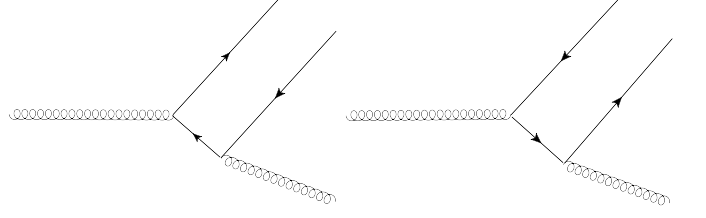


Figure 1: Feynman diagrams for the process $g \rightarrow H_c(c\bar{c})$ at LO.

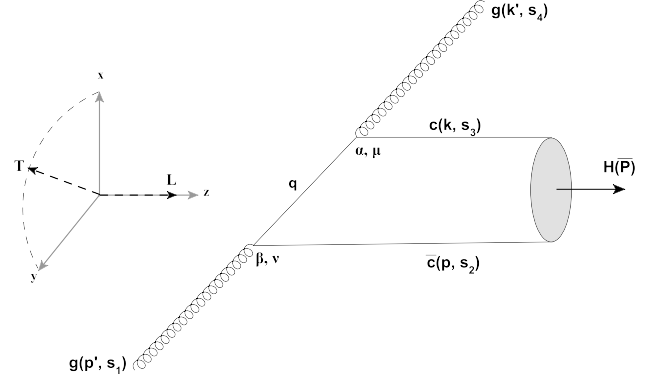


Figure 2: Formation of a heavy charmonium. A gluon forms a bound state $c\bar{c}$ with a gluon produced through a single c -quark.

be computed perturbatively from partonic subprocesses, and the distribution amplitude Φ_M contains the bound state nonperturbative dynamic of outgoing meson. This convolution is expressed as

$$T_M = \int [dx_i] T_H(x_i, Q^2) \Phi_M(x_i, Q^2), \quad (3)$$

where, $[dx_i] = dx_1 dx_2 \delta(1 - x_1 - x_2)$ and x_i 's are the momentum fractions carried by the constituent quarks. This scheme, introduced in [19, 20], is applied to absorb the soft behavior of the bound state into the hard scattering amplitude T_H . The amplitude T_H is, in essence, the partonic cross section to produce a heavy quark-antiquark ($Q\bar{Q}$) pair with certain quantum numbers that, in the old fashioned perturbation theory, is written as

$$T_H = \frac{4\pi\alpha_s m_c^2 C_F}{2\sqrt{2}P_0 k'_0 p'_0} \frac{\Gamma}{(P_0 + k'_0 - p'_0)}, \quad (4)$$

where C_F is the color factor, α_s is the strong coupling constant and Γ represents an appropriate combination of the quark propagator and spinorial parts of the amplitude as

$$\Gamma = G \left\{ \bar{u}(k, s_3) \epsilon_4^* (q + m_c) \not{\epsilon}_1 v(p, s_2) \right\}. \quad (5)$$

Here, ϵ is the polarization vector of gluon and $G = 1/(q^2 - m_c^2) = 1/(2k.k')$ is proportional to the quark propagator. In (3), Φ_M is the process-independent probability amplitude to find the quarks which are collinear in the mesonic bound state and contains the nonperturbative dynamic of the mesonic bound state. In general, the distribution amplitude is related to the mesonic wave function Ψ_M by

$$\Phi_M(x_i, Q^2) = \int [d^2 \mathbf{q}_{\perp i}] \Psi_M(x_i, \mathbf{q}_{\perp i}) \Theta(\mathbf{q}_{\perp i}^2 < Q^2), \quad (6)$$

where

$$[d^2 \mathbf{q}_{\perp i}] = 2(2\pi)^3 \delta\left[\sum_{j=1}^2 \mathbf{q}_{\perp j}\right] \prod_{i=1}^2 \frac{d^2 \mathbf{q}_{\perp i}}{2(2\pi)^3}. \quad (7)$$

In the equation above, $\mathbf{q}_{\perp i}$ refers to the transverse momentum of constituent quarks. A hadronic wave function Ψ_M which is the nonrelativistic limit of the Bethe-Salpeter equation with the QCD kernel is given in [20], so that by working in the infinite-momentum frame it can be estimated as a delta function [10]. Therefore, the distribution amplitude for a S-wave heavy meson at large Q^2 , reads [21]

$$\Phi_M \approx \frac{f_M}{2\sqrt{3}} \delta(x_1 - \frac{m_c}{M}), \quad (8)$$

where M is the heavy meson mass and $f_M = \sqrt{12/M} |\Psi(0)|$ is the meson decay constant which is related to the nonrelativistic S-wave function $\Psi(0)$ at the origin. The delta-function form is convenient for our assumption where the constituent quarks are emitted collinearly, without any transverse momentum. Substituting Eqs. (4) and (8) in (3) and carrying out the necessary integrations, the probability amplitude (3) is found as

$$T_M = \frac{\pi \alpha_s m_c^2 f_M C_F}{\sqrt{6 \bar{P}_0 k'_0 p'_0}} \frac{\Gamma}{(\bar{P}_0 + k'_0 - p'_0)}. \quad (9)$$

Now the fragmentation function (2) reads

$$D_g^{H_c}(z, \mu_0) = \frac{1}{6} [\pi \alpha_s f_M C_F m_c^2]^2 \times \int \frac{\bar{\Gamma} \Gamma \delta^3(\mathbf{k}' + \bar{\mathbf{P}} - \mathbf{p}')}{(\bar{P}_0 p'_0 k'_0)(\bar{P}_0 + k'_0 - p'_0)^2} d^3 \bar{\mathbf{P}} d^3 \mathbf{k}'. \quad (10)$$

A. kinematics

To proceed our discussion, considering Fig. 2 we need to specify our kinematics. We choose an infinite momentum frame in which the relevant four-momentum of the initial gluon is set as $p'_\mu = [p'_0; \mathbf{k}_T, p'_L]$. The transverse momentum of the initial gluon is only carried by the final state gluon which produces a jet and its

four-momentum is written as $k'_\mu = [k'_0; \mathbf{k}_T, k'_L]$. We let the produced heavy quarkonium moves in the fragmentation axis (Z -axis), which is defined as the direction of the three-momentum of the heavy quark Q in the laboratory frame after production. The four-momentum of quarkonium is set as $\bar{P}_\mu = [\bar{P}_0; \mathbf{0}, \bar{P}_L]$. The constituents of the quarkonium after creation move along the fragmentation axis and their momenta are defined as $p_\mu = [p_0; \mathbf{0}, p_L]$ and $k_\mu = [k_0; \mathbf{0}, k_L]$. We also assume that there is only one jet in the final state, since the very high momentum of the initial parton will predominantly be carried in the forward direction.

According to the definition of the fragmentation parameter $z = E^H/E^g = \bar{P}_0/p'_0$ (1), the heavy quarkonium takes a fraction z of the initial gluon energy p'_0 and the outgoing gluon takes the remaining $(1 - z)$. Therefore, the parton energies can be parametrized in terms of the initial gluon energy; $p_0 = x_1 z p'_0$, $k_0 = x_2 z p'_0$, $k'_0 = (1 - z) p'_0$, where $x_1 = p_0/\bar{P}_0$ and $x_2 = k_0/\bar{P}_0$ are the quarkonium energy fractions carried by the constituent quarks. Following Ref. [18, 22], we also assume that the contribution of each constituent quark from the meson energy is proportional to its mass, i.e. $x_1 = m_c/M$ and $x_2 = m_{\bar{c}}/M$, where M is the heavy quarkonium mass. Note that for charmonium and bottomonium states we have $m_1 = m_2 = m_Q$ and $M \cong 2m_Q$, then one has $x_1 = x_2 \cong 1/2$.

B. Unpolarized fragmentation functions

To obtain an analytical form of FF for gluon to split into the unpolarized charmonium state, in Eq. (10) one has to perform an average over the spin of the initial gluon and a sum over the colors and spins. Thus the mean amplitude squared reads

$$\overline{\Gamma^2} = \overline{|\Gamma \bar{\Gamma}|} = \frac{1}{1 + 2s_g} \sum_{s,c} \Gamma \bar{\Gamma}, \quad (11)$$

where s_g is the spin of the initial gluon. Then we obtain

$$\begin{aligned} \sum_{s,c} \Gamma \bar{\Gamma} &= G^2 \text{Tr}[(\not{k} + m_c) \gamma^\mu (\not{k}' + m_c) \gamma^\nu (\not{p}' - m_c) \\ &\quad \times \gamma_\nu (\not{k} + m_c) \gamma_\mu] = -32G^2 \left\{ 2m_c^4 + \right. \\ &\quad \left. m_c^2 [2k.k' + k.p + p.k'] - (p.k')(k.k') \right\}, \quad (12) \end{aligned}$$

where, we applied the energy projection operators

$$\begin{aligned} \Lambda^+(k) &= \sum_{S_3} u(k, s_3) \bar{u}(k, s_3) = (\not{k} + m_c), \\ \Lambda^-(p) &= \sum_{S_2} v(p, s_2) \bar{v}(p, s_2) = (\not{p} - m_c). \quad (13) \end{aligned}$$

C. Polarized fragmentation functions: First scenario

Considering $v(p)$ and $\bar{u}(k)$ as the Dirac spinors of the quarks forming the charmonium bound states, in the non-relativistic approximation the projection operator can be defined as [22, 23]

$$\Lambda_{S,S_z}(\bar{P}) = v(p)\bar{u}(k) = \frac{f_M}{\sqrt{48}}(\bar{P} - M)\Pi_{SS_z}, \quad (14)$$

where M and \bar{P} are the mass and the four-momentum of meson bound state, f_M is the meson decay constant and Π_{SS_z} is the appropriate spin projection operator; $\Pi_{00} = \gamma_5$ for pseudoscalar and $\Pi_{1S_z} = \not{\epsilon}$ for vector states. The spin content of meson is then given by either γ_5 or $\not{\epsilon}(S_z)$, as could well be expected. Therefore, the spinorial part of the amplitude (5) for formation of the pseudoscalar ($S = 0$) and the vector ($S = 1$) charmonium states may be presented in the following forms

$$\begin{aligned} \Gamma^P &\propto G \left\{ (\bar{P} - M) \gamma_5 \not{\epsilon}_4^* (q + m_c) \not{\epsilon}_1 \right\}, \\ \Gamma^V &\propto G \left\{ (\bar{P} - M) \not{\epsilon}_4^* (q + m_c) \not{\epsilon}_1 \right\}, \end{aligned} \quad (15)$$

where $q = k + k'$ is the energy-momentum of the virtual intermediate quark, ϵ is the polarization four-vector of meson which may be in a longitudinal state $\epsilon^{(L)\mu} = \epsilon^\mu(\bar{P}, \lambda = 0)$ or a transverse state $\epsilon^{(T)\mu} = \epsilon^\mu(\bar{P}, \lambda = \pm 1)$. These components satisfy the following relations

$$\begin{aligned} \epsilon(\bar{P}, \lambda) \cdot \bar{P} &= 0, \quad \epsilon(\bar{P}, \lambda) \cdot \epsilon^*(\bar{P}, \lambda') = -\delta_{\lambda, \lambda'} \\ \epsilon^{(T)} \cdot \bar{P} &= 0 = \epsilon^{(L)} \times \bar{P}. \end{aligned} \quad (16)$$

Therefore, for a quarkonium for which $\bar{P}^\mu = [\bar{P}_0; \mathbf{0}, \bar{P}_L]$, the polarization four-vector (or the spin wave function) is expressed as

$$\begin{aligned} \epsilon^{(L)\mu} &= \frac{1}{M}(\bar{P}_L; 0, 0, \bar{P}_0) = \frac{1}{M}(k_L + p_L; 0, 0, p_0 + k_0), \\ \epsilon^{(T)\mu} &= \frac{1}{\sqrt{2}}(0; \mp 1, -i, 0). \end{aligned} \quad (17)$$

Now to obtain an analytical form of FF for the polarized charmonium, in (10) we perform an average over the spin of the initial gluon and a sum over the colors and the spins of gluons. The result reads

$$|\Gamma^{V,P}|^2 = \frac{1}{1 + 2s_g} \sum_{s_g, c_g} \Gamma^{V,P} \bar{\Gamma}^{V,P}, \quad (18)$$

then we find

$$\begin{aligned} \sum_{s_g} \Gamma^P \bar{\Gamma}^P &= 128G^2 \left\{ 2m_c^4 - Mm_c^3 + 2(k.p)(k.k') \right. \\ &\quad \left. + M^2(k.k') + m_c^2[M^2 + 2k.k' + 2k.p] \right. \\ &\quad \left. - Mm_c[k.k' + k.p + p.k'] \right\}, \\ \sum_{s_g} \Gamma^V \bar{\Gamma}^V &= 128G^2 \left\{ -m_c^2M^2 + 2m_c^2(p'.k') - M^2(k.k') \right. \\ &\quad \left. + 2(k.k')(k'.p') - Mm_c[k.k' - k.p' - p'.k'] \right. \\ &\quad \left. + 2(\epsilon.k')^2 + 2(\epsilon.k)(\epsilon.k') - 2(\epsilon.k)(\epsilon.p') - \right. \\ &\quad \left. 2(\epsilon.p')(\epsilon.k') \right\}. \end{aligned} \quad (19)$$

D. Polarized fragmentation functions: Second scenario

Following Ref. [24], in the nonrelativistic approximation the spin projection operator can be also defined as

$$\begin{aligned} \Lambda_{S,S_z}(p, k) &= \\ &\sqrt{\frac{3}{m}} \sum_{S_2, S_3} \left\langle \frac{1}{2} S_2; \frac{1}{2} S_3 | S, S_z \right\rangle v(p+q) \bar{u}(k-q), \end{aligned} \quad (20)$$

where $\langle S_2/2; S_3/2 | S, S_z \rangle$ are the Clebsch-Gordan coefficients and S_2, S_3 and q are the spins and the relative four-momentum of the constituent quarks, respectively. These matrices can be written in a simple covariant form up to terms which are of q^2 -order in the quarkonium rest frame. Such terms are irrelevant in the nonrelativistic approximation which we are adopting. Therefore one finds [24],

$$\Lambda_{S,S_z}(p, k) = \sqrt{\frac{3}{8m^2}} [(\not{k} - \not{q} + m_2) \Pi_{S,S_z} (-\not{p}' - \not{q}' + m_1)], \quad (21)$$

where Π_{S,S_z} is as before. Ignoring the Fermi motion, so that the constituent quarks will fly together in parallel, Eq. (21) can be simplified as

$$\Lambda_{S,S_z}(p, k) \propto (\not{k} + m_2) \Pi_{S,S_z} (\not{p}' - m_1). \quad (22)$$

In our calculation $m_1 = m_2 = m_c$. Thus, the amplitude squared $\Gamma \bar{\Gamma}$ (18) in the second scenario is given by

$$\begin{aligned} \sum_{s_g} \Gamma^{V,P} \bar{\Gamma}^{V,P} &= G^2 \text{Tr} \left[(\not{k} + m_c) \Pi_{S,S_z} (\not{p}' - m_c) \gamma^\mu (\not{q} + m_c) \right. \\ &\quad \left. \times \gamma^\nu \gamma_\nu (\not{q} + m_c) \gamma_\mu (\not{p}' - m_c) \Pi_{S,S_z} (\not{k} + m_c) \right], \end{aligned} \quad (23)$$

where $\Pi_{S,S_z} = \gamma_5$ for $c\bar{c}(^1S_0)$ and $\Pi_{S,S_z} = \not{\epsilon}$ for $c\bar{c}(^3S_1)$. Using the traditional trace technique, the necessary traces are evaluated and the results are expressed as the dot products of four-vectors.

E. Polarized fragmentation functions: Third scenario

In the third scenario the spin projection operator is defined as

$$\Lambda_{S,S_z}(p, k) \propto (\not{p} + m_c) \Pi_{S,S_z}. \quad (24)$$

This scenario is defined in [24] and applied in [11] to obtain the spin dependent fragmentation functions for a heavy quark Q to fragment into vector $Q\bar{Q}({}^3S_1)$ and pseudoscalar $Q\bar{Q}({}^1S_0)$ heavy quarkonium to leading order perturbative QCD. Therefore,

$$\begin{aligned} \sum_{s_g} \Gamma^{V,P} \bar{\Gamma}^{V,P} &= G^2 Tr \left[(\not{p} + m_c) \Pi_{S,S_z} \gamma^\mu (\not{q} + m_c) \gamma^\nu \gamma_\nu \right. \\ &\quad \left. \times (\not{q} + m_c) \gamma_\mu \Pi_{S,S_z} (\not{p} + m_c) \right]. \end{aligned} \quad (25)$$

Now, the spinorial part of the amplitude to produce the pseudoscalar and vector charmonium states read as

$$\begin{aligned} \sum_{s_g} \Gamma^P \bar{\Gamma}^P &= 128 G^2 m_c^2 \left\{ 2m_c^2 + 2(k.k') + k.p + p.k' \right\}, \\ \sum_{s_g} \Gamma^V \bar{\Gamma}^V &= -128 G^2 \left\{ 2(k.k') \left[m_c^2 - p'.k' - k.p' + k.k' \right] \right. \\ &\quad + m_c^2 \left[m_c^2 - 2(k.\epsilon)^2 - 2(k'.\epsilon)^2 - k.p' - k'.p' \right. \\ &\quad \left. \left. + 2(k'.\epsilon)(p'.\epsilon) + (k.\epsilon) \left(2p'.\epsilon - 4k'.\epsilon \right) \right] \right\}. \end{aligned} \quad (26)$$

III. ANALYTICAL RESULTS

To obtain the FFs for unpolarized and polarized quarkonia, using the kinematics, first we put the dot products of the relevant four-vectors in the following forms:

$$\begin{aligned} 2k.p &= \frac{M^2}{8}, \\ 2p'.k' &= \frac{z^2}{1-z} k_T^2, \\ 2\epsilon_L.k &= 2\epsilon_L.p = 0, \\ 2\epsilon_L.p' &= -\frac{M}{z} + \frac{zk_T^2}{M}, \\ 2\epsilon_T.p' &= -\sqrt{2}k_T(-1+i), \\ 2\epsilon_T.k' &= -\sqrt{2}k_T(-1-i), \\ 2k.p' &= 2p.p' = \frac{zm}{M} k_T^2 + \frac{mM}{z}, \\ 2\epsilon_L.k' &= -\frac{M(1-z)}{z} + \frac{zk_T^2}{M(1-z)}, \\ 2p.k' &= 2k.k' = \frac{zm}{(1-z)M} k_T^2 + \frac{mM(1-z)}{z}. \end{aligned} \quad (27)$$

To perform the phase space integrations (10), first we consider the following integral

$$\int \frac{d^3\bar{\mathbf{P}} \delta^3(\bar{\mathbf{P}} + \mathbf{k}' - \mathbf{p}')}{\bar{P}_0(\bar{P}_0 + k'_0 - p'_0)^2} = \frac{\bar{P}_0}{(M^2 + 2p'.k')^2}, \quad (28)$$

and instead of performing the transverse momentum integration we replace the integration variable by its average value $\langle k_T^2 \rangle$, which is a free parameter and can be specified experimentally. Therefore we can write

$$\begin{aligned} \int F(z, k_T^2) d^3k' &= \int F(z, k_T^2) dk'_L d^2k'_T \\ &\approx m_c^2 k'_0 F(z, \langle k_T^2 \rangle). \end{aligned} \quad (29)$$

In conclusion, considering the contributions of both Feynman diagrams in Fig. 1, we obtain the *unpolarized* FF for $g \rightarrow H_c(c\bar{c})$ at the initial scale $\mu_0 = 2m_c$ as

$$\begin{aligned} D_{g \rightarrow H_c}(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} \times \\ &\quad \left\{ \left[\frac{z^2 k_T^2 + M^2(1-z)^2}{Mz(1-z)} - 3m_c^2 \right]^2 - 21m_c^2 \right\}, \end{aligned} \quad (30)$$

where,

$$g(z, \langle k_T^2 \rangle) = \left[M^2 + \frac{z^2 k_T^2}{1-z} \right]^2 \left[\frac{z^2 k_T^2 + M^2(1-z)^2}{Mz(1-z)} \right]^2, \quad (31)$$

and N is proportional to $(\pi C_F m_c^3 \alpha_s f_M)^2$ but it is related to the normalization condition [8, 9].

The *polarized* FFs, using the *first scenario* are written as

$$\begin{aligned} D_g^P(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} (64m_c) \left[3m_c^2 + (M - m_c)^2 \right] \\ &\quad \times \left[2m_c + \frac{zk_T^2}{M(1-z)} + \frac{M(1-z)}{z} \right], \\ D_g^T(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} (64m_c) \left[\frac{z^3 k_T^4}{M(1-z)^2} - m_c M^2 \right. \\ &\quad \left. - M^3 \frac{1-z}{z} + k_T^2 (4M + \frac{m_c}{1-z} z^2) \right], \\ D_g^L(z, \mu_0) &= \\ &\quad \frac{Nz}{g(z, \langle k_T^2 \rangle)} \frac{64m_c}{z} \left[\frac{z^2 k_T^2}{(1-z)} (m_c - M) - m_c M^2 \right]. \end{aligned} \quad (32)$$

Note that, the fragmentation function for a vector charmonium $H_c({}^3S_1)$ is the sum of the longitudinal and twice the transverse components, i.e.,

$$D_g^V(z, \mu_0) = 2D_g^T + D_g^L, \quad (33)$$

where T and L refer to the longitudinal and the transverse polarization of vector charmonium.

The polarized FFs, using the *second scenario* may be written as

$$\begin{aligned}
D_g^P(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} (768m_c^5) \\
&\times \left[\frac{zk_T^2}{M(1-z)} + 2m_c - \frac{M(z-1)}{z} \right], \\
D_g^T(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} \frac{64m_c}{z} \left[\frac{z^6 k_T^6 (m_c - M)^2}{M^3(z-1)^3} \right. \\
&+ \frac{z^2 k_T^4}{M^2(1-z)^2} \left\{ -6z^2 m_c^3 + Mm_c^2(z^3 + 11z^2 - 12z \right. \\
&+ 8) - 2m_c M^2 z(z^2 + 2z - 2) + z^2 M^3(z-1) \left. \right\} + \\
&\frac{m_c k_T^2}{M(z-1)} \left\{ -2M^3 z^2(z-1) + 4z^2 m_c^3(1+2z) - \right. \\
&2Mm_c^2 z(7z^2 - 8z + 8) + m_c M^2(8z^3 - 11z^2 \\
&+ 12z - 8) \left. \right\} - m_c^2 \left\{ 8zm_c^3 + 4Mm_c^2(1-3z) + \right. \\
&6m_c M^2(z-1) - M^3(z-1) \left. \right\} \left. \right], \\
D_g^L(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} \frac{64m_c}{z} \left[\frac{z^6 k_T^6}{M^3(z-1)^3} \left\{ M^2 + \right. \right. \\
&(z-3)m_c(M-m_c) \left. \right\} - \frac{z^4 k_T^4}{M^2(1-z)^2} \left\{ 2m_c^3(z+3) \right. \\
&- Mm_c^2(2z+5) + 3zm_c M^2 + M^3(1-z) \left. \right\} + \\
&\frac{m_c z^2 k_T^2}{M(z-1)} \left\{ (8z+4)m_c^3 - 10Mm_c^2 z + \right. \\
&M^2 m_c(7z-5) + M^3(3-2z) \left. \right\} - m_c^2 \left\{ 8zm_c^3 + \right. \\
&4Mm_c^2(1-3z) + 2M^2 m_c(4z-3) + M^3(3-2z) \left. \right\} \left. \right]. \tag{34}
\end{aligned}$$

And the polarized FFs in the *third scenario* are expressed as

$$\begin{aligned}
D_g^P(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} (192m_c^3) \\
&\times \left[\frac{zk_T^2}{M(1-z)} + 2m_c - \frac{M(z-1)}{z} \right], \\
D_g^T(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} \frac{64m_c}{z} \left[\frac{z^4 k_T^4 (M-m_c)}{M^2(z-1)^2} \right. \\
&+ \frac{zk_T^2}{M(z-1)} \left\{ -2Mm_c((1+z)^2 - 3) \right. \\
&+ M^2 z(z-1) + m_c^2 z(1+z) \left. \right\} - m_c \left\{ \right. \\
&2zm_c^2 + M^2(z-1) + Mm_c(1-2z) \left. \right\} \left. \right], \tag{35}
\end{aligned}$$

and

$$\begin{aligned}
D_g^L(z, \mu_0) &= \frac{Nz}{g(z, \langle k_T^2 \rangle)} \frac{64m_c}{z} \left[\frac{z^4 k_T^4}{M(z-1)^2} + \right. \\
&\frac{z^2 k_T^2}{M(z-1)} \left\{ m_c^2(1+z) - 3m_c Mz + \right. \\
&M^2(z-1) \left. \right\} - m_c^2 \left(M + 2z(m_c - M) \right) \left. \right]. \tag{36}
\end{aligned}$$

We are now in a position to present our phenomenological predictions for the gluon fragmentation into the spin-singlet and -triplet S-wave charmoniums, by performing a numerical analysis. In general, fragmentation function $D_{g \rightarrow H_c}(z, \mu^2)$ depends on both the fragmentation parameter z and factorization scale μ . The scale μ is generally arbitrary, but in a high energy process where a jet is produced with transverse momentum k_T , large logarithms of k_T/μ in the partonic cross section (perturbative part of the hadroproduction) can be avoided by choosing μ on the order of k_T . The functions (30, 32, 34-36) should be regarded as models for the gluon fragmentation into the polarized and unpolarized charmoniums at a scale μ of order m_Q . For values of μ much larger than μ_0 , the initial FFs should be evolved from the scale μ_0 to the scale μ using the Altarelli-Parisi equations [25–27].

For numerical results, we take $m_c = 1.5$ GeV, $\alpha_s(2m_c) = 0.26$ and $f_M(c\bar{c}) = 0.48$ GeV [28]. The initial scale of fragmentation is set to $\mu_0 = 2m_c$ and the transverse momentum of the initial gluon is selected as $\langle k_T \rangle = 10$ GeV. In Fig. 3, using the first scenario our predictions for the polarized charmonium FFs, i.e. $g \rightarrow H_c(^1S_0, ^3S_1)$, and the unpolarized one are shown. Here, D_V is the convenient summation of the longitudinal and transverse fragmentation functions, see Eq. (33). In Fig. 4, using the second scenario the behavior of the fragmentation functions for the longitudinally and transversely components of vector meson are shown. Those are also compared with the pseudoscalar and the unpolarized ones, while for the unpolarized case a summation is going over the spin of the constituent heavy quarks in which case we apply the energy projection operators defined in (13). The fragmentation functions shown in Fig. 4 are also compared in Fig. 5 by taking $D^V = 2D^T + D^L$. In Fig. 6, the same comparison is done but using the third scenario. As is seen, in all scenarios the fragmentation probability of the gluon into the spin-triplet charmonium is the biggest one. The position of the fragmentation peak is almost the same in all scenarios but the third scenario shows a bigger peak.

Our result shown in Fig. 6, is in reasonable agreement with the result presented in Fig. 3 of Ref. [29], when the non-covariant definition of fragmentation parameter (1) is applied. In both results, the peak position of the fragmentation function occurs at $z \approx 0.22$ when $p_T = 10$ GeV is considered, and the maximum value of the vector charmonium FF is $D_g^V \approx 0.8 \times 10^{-5}$. Note that in [29], authors have applied the Braaten's approach which

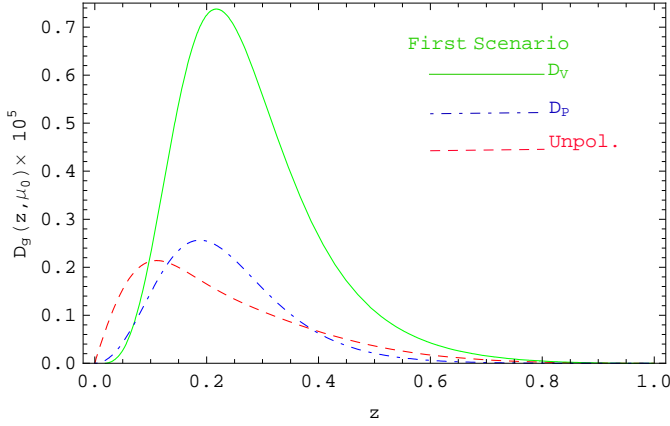


Figure 3: The fragmentation functions for the vector (solid line) and the pseudoscalar (dot-dashed line) charmoniums in the first scenario. For comparison the unpolarized one (dashed line) is also shown. The initial scale is $\mu_0 = 2m_c$

is completely different with the scheme used in our work. Besides the $g \rightarrow H_c$ FF itself, also its first moment is of phenomenological interest. It corresponds to the $g \rightarrow H_c$ branching fraction

$$B(\mu) = \int_0^1 dz D_g^{H_c}(z, \mu). \quad (37)$$

Using the third scenario, our result for the $g \rightarrow H_c(^3S_1)$ branching fraction is $B(2m_c) = 3.01 \times 10^{-6}$ which can be compared with the result presented in [1] where $B(2m_c) = 3.2 \times 10^{-6}$.

By these comparisons, it seems that the third scenario is more suitable than the other ones. However, the advantage of the projection operator in the first scenario is that it defines the bound state mass of heavy meson in our calculations.

Our results can be directly applied to the S-wave bottomonium states $\Upsilon(^3S_1)$ and $\eta_b(^1S_0)$, except that m_c is replaced by $m_b = 4.5$ GeV and the decay constant $f_M(b\bar{b}) = 0.33$ GeV [28] is the appropriate constant for the bottomonium mesons. Since the b-quark is heavier than the c-quark, the peaks of the fragmentation functions shift significantly toward higher values of z .

As we explained, the fragmentation is related to the low-energy part of the hadron production so, in principle, should be extracted from experimental data. However, in practice, it is hard to extract FFs phenomenologically, especially for the heavy quarkonia that, at the present, there are no experimental data for them. A comprehensive review on the behavior of the gluon FFs obtained phenomenologically, shows that for the heavy-light mesons (e.g. B, D) the peaks of the gluon FFs occur at low values of the fragmentation parameter z and the FFs approach to zero when either $z \rightarrow 0$ or $z \rightarrow 1$. For example, the behavior of the $g \rightarrow B$ FF is shown in [30] and the $g \rightarrow D^+$ one is shown in [31]. These behaviors

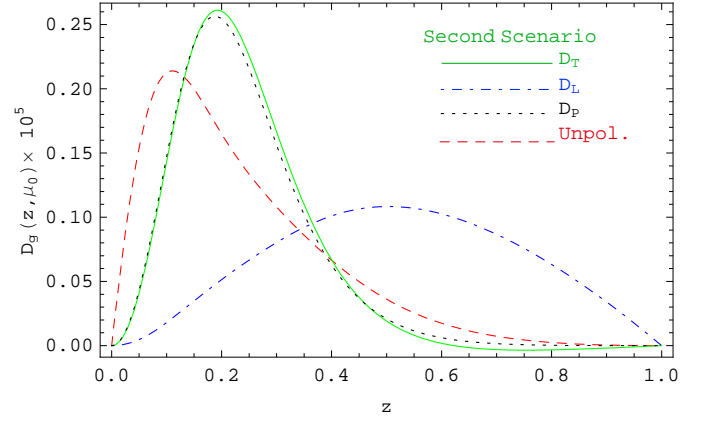


Figure 4: The longitudinal (dot-dashed line) and transverse (solid line) components of the vector charmonium fragmentation function in the second scenario. The pseudoscalar (dots) and the unpolarized (dashed line) FFs are also shown.

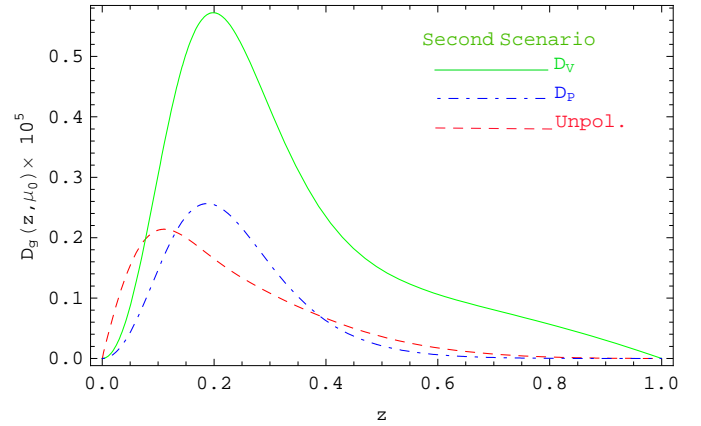


Figure 5: As in Fig. 3 but in the second scenario.

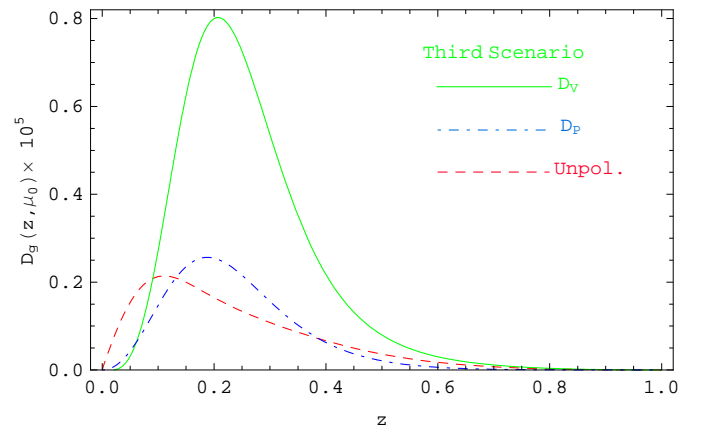


Figure 6: As in Fig. 3 but in the third scenario.

may be expected for the heavy quarkonia, as shown in Figs. 3-6.

In Ref. [32], we pointed out that in top-quark decays $t \rightarrow b + W^+ (+g)$, since the observed mesons can be also produced through a fragmenting gluon, therefore, to obtain the most accurate result for the energy spectrum of the meson, one has to add the contribution of gluon fragmentation to the b-quark to produce the outgoing meson. We showed that the $g \rightarrow B/D$ contribution is appreciable only at a low energy of the observed meson so the contribution of the gluon leads to an appreciable reduction in decay rate at low values of the meson energy, which is in consistency with the peak position of the gluon FF. However, the contribution of the gluon cannot be discriminated so, this part of calculation is of more theoretical relevance and there will be no experimental data for the $g \rightarrow B/D$. But the $g \rightarrow B/D/H_c$ FFs are confirmed by the comparison between the theoretical predictions and the experimental measurements of the heavy meson cross sections at the LHC.

IV. CONCLUSION

The dominant mechanism to produce the heavy quarkonia at high transverse momentum is fragmentation; the production of a high energy parton followed by its fragmenting into the heavy quark-antiquark bound states. Beside the phenomenological approaches, there are some theoretical models to calculate the fragmenta-

tion functions analytically. In the present work, using the Suzuki's model we gave out an analytical expression for the initial scale fragmentation functions of gluon to split into the spin-singlet and -triplet S-wave charmonium states. Our results depend on the transverse momentum of the initial gluon whereas in other models the integrations over all freedom degrees are performed. Since the transverse momentum dependent FFs show up explicitly in several semi-inclusive cross sections, therefore in the QCD corrections the inclusion of these dependent FFs will be necessary.

We discussed that there are three different scenarios to impose the spin effects of heavy quarkonia into the gluon FFs and we presented the fragmentation functions in each three scenarios. The results show that the fragmentation probability of the gluon into the spin-triplet bound-state is the biggest one. However, at present there are no experimental data for the heavy quarkonium fragmentation but a comparison between the treatment of the gluon FFs in Figs. 3, 5 and 6 with the $g \rightarrow B$ [30] and $g \rightarrow D$ [31] FFs can confirm the correctness of our derived results. Our results can be directly used for the S-wave bottomonium states ($b\bar{b}$ -system), with some simple replacements.

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